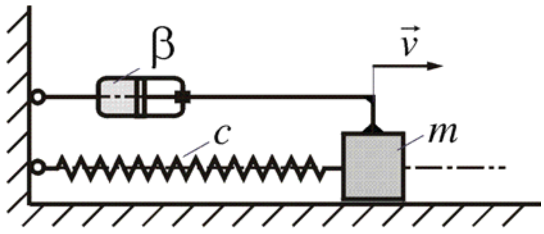
GRAĐEVINSKI FAKULTET UNIVERZITETA U BEOGRADU

Ispit iz **TEHNIČKE MEHANIKE 2,** **GRUPA A** **29.01.2007.**

1.ZADATAK: (20%)

Telo zanemarljivih dimenzija mase *m,* vezano oprugom krutosti *c*, može da se kreće po glatkoj horizontalnoj podlozi. Osim opruge za telo je vezan i klip sa cilindrom. Sila otpora koja dejstvuje na telo koje se kreće, po intenzitetu je jednaka $R = \beta v^2$, gde je v intenzitet brzine klipa (odnosno posmatranog tela), a koeficijent $\beta = const > 0$. Odrediti intenzitet početne brzine tela v_0, pri kome će maksimalno udaljenje tereta od ravnotežnog položaja biti jednako *L*.



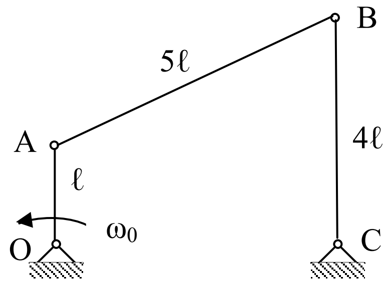
Slika 1

2.ZADATAK: (20%)

Štap OA zglavkastog četvorougla OABC obrće se konstantnom ugaonom brzinom ω_0. Odrediti:

a) ugaone brzine štapova AB i BC, kao i brzinu tačaka A i B.

b) ugaono ubrzanje štapa AB i intenzitet ubrzanja tačke B, u položaju mehanizma prikazanog na slici. Dato je: OA=ℓ, AB=5ℓ, BC=4ℓ.



<u>NAPOMENA:</u> Uslov za polaganje ispita je:

- min. 20% (od 40%) na 1. i 2. zadatku,
- min. 30% (od 60%) na 3. i 4. zadatku.

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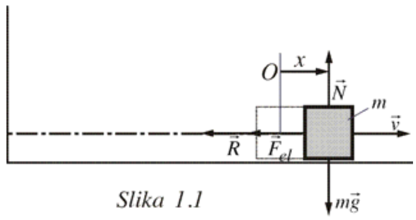
R E Š E NJ A

1.ZADATAK: (20%)



Slika 1.1

$F_{el} = cx, \quad R = \beta \dot{x}^2$,

$$m\ddot{x} = -cx - \beta\dot{x}^2, \qquad \ddot{x} = -\frac{c}{m}x - \frac{\beta}{m}\dot{x}^2, \qquad \frac{\dot{x}d\dot{x}}{dx} + \frac{\beta}{m}\dot{x}^2 = -\frac{c}{m}x, \qquad \frac{1}{2}\frac{d\dot{x}^2}{dx} + \frac{\beta}{m}\dot{x}^2 = -\frac{c}{m}x,$$

$$\dot{x}^2 = u \quad \Rightarrow \quad \frac{1}{2}u' + \frac{\beta}{m}u = -\frac{c}{m}x, \; u(x) = u_h + u_p,$$

homogeni deo: $\frac{1}{2}u' + \frac{\beta}{m}u = 0, \quad u_h = Ce^{\lambda x}, \quad u'_h = C\lambda e^{\lambda x}$,

$$\frac{1}{2}\lambda + \frac{\beta}{m} = 0, \quad \Rightarrow \quad \lambda = -\frac{2\beta}{m} \quad \Rightarrow \quad u_h = Ce^{-\frac{2\beta}{m}x},$$

partikularni deo: $u_p = Ax + B \quad \Rightarrow \quad u'_p = A,$

$$\frac{1}{2}A + \frac{\beta}{m}Ax + \frac{\beta}{m}B = -\frac{c}{m}x \quad \Rightarrow \quad \frac{1}{2}A + \frac{\beta}{m}B = 0, \quad \frac{\beta}{m}A = -\frac{c}{m} \quad \Rightarrow \quad A = -\frac{c}{\beta}, \quad B = \frac{1}{2}\frac{cm}{\beta^2},$$

$$u(x) = u_h + u_p = Ce^{-\frac{2\beta}{m}x} - \frac{c}{\beta}x + \frac{1}{2}\frac{cm}{\beta^2}$$

$$\dot{x}^2 = v^2 = Ce^{-\frac{2\beta}{m}x} - \frac{c}{\beta}x + \frac{1}{2}\frac{cm}{\beta^2}, \quad za \;\; t_0 = 0, \;\; x_0 = 0, \;\; v_0, \quad \Rightarrow \quad v_0^2 = C + \frac{1}{2}\frac{cm}{\beta^2} \quad \Rightarrow \quad C = v_0^2 - \frac{1}{2}\frac{cm}{\beta^2},$$

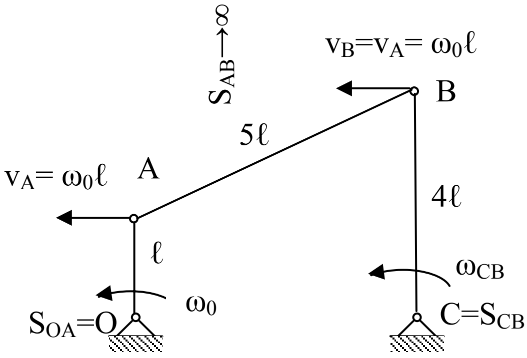
$$v^2 = \left(v_0^2 - \frac{1}{2}\frac{cm}{\beta^2}\right)e^{-\frac{2\beta}{m}x} - \frac{c}{\beta}x + \frac{1}{2}\frac{cm}{\beta^2},$$

Za maksimalno udaljenje od ravnotežnog položaja važi

$$v_1 = 0, \quad x_1 = L \quad \Rightarrow \quad 0 = \left(v_0^2 - \frac{1}{2}\frac{cm}{\beta^2}\right)e^{-\frac{2\beta}{m}L} - \frac{c}{\beta}L + \frac{1}{2}\frac{cm}{\beta^2} \quad \Rightarrow \quad v_0 = \sqrt{\frac{cm}{2\beta^2}\left[1 + \left(\frac{2\beta L}{m} - 1\right)e^{\frac{2\beta L}{m}}\right]}$$

2.ZADATAK: (20%)

Brzine:



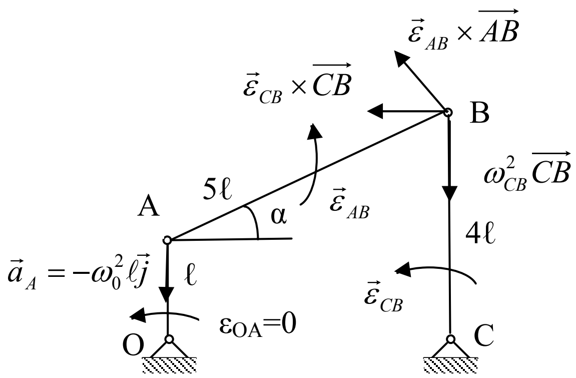
$v_A=\omega_0\ell$

$v_B=v_A=\omega_0\ell$

$\omega_{CB}=\omega_0/4$

$\omega_{AB}=0$

Ubrzanja:



$$\vec{a}_A=\underbrace{\vec{a}_0}_{0}+\underbrace{\vec{\varepsilon}_1}_{0}\times\overrightarrow{OA}-\omega_0^2\overrightarrow{OA}\Rightarrow\vec{a}_A=-\omega_0^2\ell\vec{j}$$

$$\left.\begin{aligned}\vec{a}_B&=\vec{a}_A+\vec{\varepsilon}_{AB}\times\overrightarrow{AB}-\underbrace{\omega_{AB}^2}_{0}\overrightarrow{AB}\\ \vec{a}_B&=\underbrace{\vec{a}_C}_{0}+\vec{\varepsilon}_{CB}\times\overrightarrow{CB}-\omega_{CB}^2\overrightarrow{CB}\end{aligned}\right\}\Rightarrow$$

$$\vec{a}_A+\vec{\varepsilon}_{AB}\times\overrightarrow{AB}=\vec{\varepsilon}_{CB}\times\overrightarrow{CB}-\omega_{CB}^2\overrightarrow{CB}\,/\,\vec{i},\vec{j}$$

$$(1)\cdot\cdot\,0-\varepsilon_{AB}\cdot 5\ell\cdot\frac{3}{5}=-\varepsilon_{CB}\cdot 4\ell$$

$$(2)\cdot\cdot-\omega_0^2\ell+\varepsilon_{AB}\cdot 5\ell\cdot\frac{4}{5}=-\frac{\omega_0^2}{16}\cdot 4\ell\ \Rightarrow\ \varepsilon_{AB}=\frac{3}{16}\omega_0^2$$

$$(1)\Rightarrow\varepsilon_{CB}=\frac{9}{64}\omega_0^2$$

$$\vec{a}_B=\underbrace{\vec{a}_C}_{0}+\vec{\varepsilon}_{CB}\times\overrightarrow{CB}-\omega_{CB}^2\overrightarrow{CB}=-\frac{9}{64}\omega_0^2\cdot 4\ell\cdot\vec{i}-\frac{\omega_0^2}{16}\cdot 4\ell\cdot\vec{j}\Rightarrow\left|\vec{a}_B\right|=\ell\omega_0^2\sqrt{\frac{81}{16^2}+\frac{1}{16}}=0.615\ell\omega_0^2$$